

Sheet 2

$$\textcircled{1} \nabla \cdot \vec{f} = 0$$

$$\begin{aligned} \nabla \cdot \vec{f} &= \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z} \\ &= -2y \sin(2x+y^2) + 2y \sin(2x+y^2) + 0 = 0 \end{aligned}$$

Then \vec{f} is solenoidal

$$\textcircled{5} F(x, y, z) = x^3 y z \vec{i} + x y^3 z \vec{j} + x y z^3 \vec{k}$$

$$\text{curl } \vec{F} = \nabla_x \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^3 y z & x y^3 z & x y z^3 \end{vmatrix}$$

$$= (x z^3 - x y^3) \vec{i} - (y z^3 - x^3 y) \vec{j} + (y^3 z - x^3 z) \vec{k}$$

~~$$\nabla \cdot (\nabla_x F)$$~~

$$\begin{aligned} \nabla \cdot (\nabla_x F) &= \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z} \\ &= z^3 - y^3 - z^3 + x^3 + y^3 - x^3 = \text{zero} \end{aligned}$$

Then $\nabla_x F$ is solenoidal

$$[4] \vec{F} = 2xy^2z \vec{i} + 2x^2yz \vec{j} + x^2y^2 \vec{k}$$

$$\vec{\nabla} \cdot \vec{F} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2xy^2z & 2x^2yz & x^2y^2 \end{vmatrix}$$

$$= (2x^2y - 2x^2y) - (2xy^2 - 2xy^2) + (4xyz - 4xyz) = 0$$

Then \vec{F} is irrotational

prove $\vec{F} = -\vec{\nabla} \phi$

electric field

electric potential

$$(F_1 \vec{i} + F_2 \vec{j} + F_3 \vec{k}) = \left(\frac{\partial \phi}{\partial x} \vec{i} + \frac{\partial \phi}{\partial y} \vec{j} + \frac{\partial \phi}{\partial z} \vec{k} \right)$$

$$\phi_x = -2xy^2z \xrightarrow{\text{integral}} \phi = -x^2y^2z + A_1(y, z)$$

$$\phi_y = -2x^2yz \longrightarrow \phi = -x^2y^2z + A_2(x, z)$$

$$\phi_z = -x^2y^2 \longrightarrow \phi = -x^2y^2z + A_3(x, y)$$

$$\phi = -x^2y^2z$$

$$\therefore \vec{F} = -\vec{\nabla} \phi$$

(5) $\phi = x^2 + y^2 - 2z^2$ prove $\nabla \cdot (\nabla \phi) = 0$

$$\nabla \phi = 2x\vec{i} + 2y\vec{j} - 4z\vec{k}$$

$$\nabla \cdot (\nabla \phi) = \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z}$$

$$= 2 + 2 - 4 = 0$$

(6) $\phi = xyz^2$

$$\nabla \phi = yz^2\vec{i} + xz^2\vec{j} + 2xyz\vec{k}$$

$$\nabla \cdot (\nabla \phi) = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz^2 & xz^2 & 2xyz \end{vmatrix}$$

$$= (2xz - 2xz) - (2yz - 2yz) + (z^2 - z^2)$$

$$= 0$$